Multiple Equilibria in a Single-Column Model of the Tropical Atmosphere

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A single-column model run under the weak temperature gradient approximation, a parameterization of large-scale dynamics appropriate for the tropical atmosphere, is shown to have multiple stable equilibria. Under conditions permitting persistent deep convection, the model has a statistically steady state in which such convection occurs, as well as an extremely dry state in which convection does not occur. Which state is reached depends on the initial moisture profile.

1. Introduction

In the extratropics, balanced dynamics associated with baroclinic eddies can force precipitation strongly by inducing ascent and adiabatic cooling. In the tropics, the reasons for the onset of precipitation at a given time and place are often much more subtle, and large-scale ascent more a response to deep convection than a cause. On the other hand, in the tropics the relationship of time-mean precipitation to boundary conditions is generally stronger than it is in the extratropics. In simple models of tropical climate dynamics, it is typical to treat the time-mean tropical precipitation as deterministically related to the boundary conditions. While this may for many purposes be adequate, it is clear that the time mean state, particularly in climatologically rainy regions, is an average over periods of strong precipitation and periods of negligible precipitation. As precipitation cannot be negative, this implies an inherent nonlinearity that is overlooked in deterministic models of the time mean state. It also suggests that transient disturbances, or other factors whose relationship to the boundary conditions is at best hard to discern, may play a role in maintaining the time-mean state.

In this study we present a very simple expression of these properties of the tropical atmosphere, namely its nonlinearity and the potential complexities hidden in its time-mean response to boundary conditions. We show that a single-column model with boundary conditions permitting strong deep convection can have two stable steady (or statistically steady) states: one in which persistent deep convection occurs, and one in which it does not. Which solution is reached in numerical integrations depends on the initial moisture field.

2. Model and Experiment Design

We use the single column version of the GEOS-5 general circulation model (GCM). A description of the GEOS-5 system and its physical parameterizations may be found at http://gmao.gsfc.nasa.gov/systems/geos5/, and a brief outline is presented in the auxiliary materials.

The model is run over an ocean surface with fixed SST. The insolation and solar zenith angle are held constant at values of 400 Wm^{-2} and zero respectively. The surface wind speed, used in the bulk formulae for the surface fluxes, is set to a constant $7ms^{-1}$.

The experiment design is essentially the same as that used in *S*obel and Bretherton [2000; hereafter SB00]. The model is first run to a state of radiative-convective equilibrium (RCE) over an SST of 301 K. In this calculation the large-scale vertical velocity is set to zero. The temperature profile from the RCE calculation is then used as an input to a set of calculations in which the model is modified to implement the weak temperature gradient (WTG) approximation. In the WTG calculations, the temperature profile is held fixed in time in the free troposphere, defined somewhat arbitrarily as those levels with pressures less than $p_{\rm inv}$= 850 hPa. At those levels, the large-scale vertical velocity is diagnosed as that which causes the vertical advection of potential temperature to precisely balance the diabatic heating computed by the model physics, consistent with the requirement of zero temperature tendency. Horizontal advection of temperature is also assumed negligible. In the nominal boundary layer, defined as levels with pressures greater than $p_{\rm inv}$, the temperature is determined prognostically, with the vertical velocity computed by linear interpolation in pressure between the diagnosed value at $p_{\rm inv}$ and an assumed value of zero at the surface. At all levels, large-scale vertical advection of humidity and condensate —

both of which are integrated prognostically as usual — are computed using the vertical gradients derived from the internally predicted profiles of those variables and the large-scale vertical velocity diagnosed as described above.

In some experiments, the initial moisture profile is taken from the steady-state RCE solution. In others, the initial moisture profile is obtained by setting the free-tropospheric humidity to zero.

In some experiments, horizontal advection of moisture is neglected, as in SB00. In others, it is parameterized by a relaxation back to a fixed profile - the steady-state RCE solution - with a fixed time scale. The time scale can be thought of as an advective one given by a length scale associated with horizontal moisture gradients divided by a velocity, where the velocity is thought of as a rotational one which is independent of height (so that the time scale itself is independent of height), and also independent of the magnitude of the divergent circulation implied by the vertical velocity in the column. Another approach is to consider advection by that divergent circulation itself, in which case the relaxation rate is simply $d\omega/dp$, where ω is the pressure vertical velocity and p the pressure [*Raymond and Zeng*, 2005]. We have performed calculations with this method also; the results are not qualitatively different from those using the fixed advective time scale.

All simulations are run for one year, by which time the model has reached a solution that is statistically steady, and in many cases close to truly steady; oscillations sometimes occur, as described briefly below. Results presented are averages over the last 2 months.

3. Results

3.1. GEOS 5 SCM

Fig. 1 shows the temperature and relative humidity profiles obtained from the RCE calculation. The temperature profile is close to moist adiabatic as expected. Fig. 2 shows time-mean precipitation as a function of SST for the WTG experiments. The figure is similar to fig. 4 of SB00; each point represents a different experiment, each of which uses the same free-tropospheric temperature profile, obtained from the RCE calculation. The difference here [besides that we use a different model; SB00 used the model of *Rennó et al.* [1994]] is that we show several sets of curves, obtained using different initial moisture profiles, and different time scales for horizontal moisture advection (SB00 did not include this process in any of their calculations). We first describe the solid curves, which were obtained using no horizontal moisture advection.

When the initial moisture profile is taken from the RCE, a rainy state is obtained for sufficiently large SST. The shape of the curve of precipitation vs. SST for this set of solutions (the upper blue curve) is qualitatively similar to that obtained by SB00. The lower blue curve curve shows solutions obtained using an initial moisture profile is set to zero in the free troposphere.

When the initial profile is sufficiently dry, apparently a dry state can be maintained although the SST is high and the convective available potential energy (CAPE) for an undilute parcel ascent would be substantial. For a dry profile, the convective parameterization is unable to generate significant convective heating or precipitation, presumably due to inhibition by entrainment of the dry air. In the absence of convective heating, radiative cooling forces descent, which maintains the dry state in the free troposphere. On the other hand if the initial conditions are sufficiently moist, for sufficiently large SST the deep convective scheme is able to become and remain active. If the ensuing heating exceeds the radiative cooling, it induces large-scale ascent, which moistens the atmosphere further by large-scale advection, leading to the maintenance of the convective state. For SST below a critical value, here around 300.5 K, there is insufficient CAPE (or strictly, work function; Arakawa and Schubert 1974) for deep convection to occur even if the initial profile is moist. Dry air can suppress deep convection in the presence of large CAPE, but moist air cannot cause it to occur if CAPE is absent. Thus the multiple equilibria occur only for SST above the critical value.

Fig. 3 shows vertical profiles of relative humidity as functions of SST in the dry state and the moist state, without horizontal moisture advection. We see that the relative humidity in the boundary layer is similar in both sets of solutions, but that the free-tropospheric humidity is essentially zero in the dry case; in the absence of horizontal moisture advection, there is no source of moisture to balance drying due to descent. This is somewhat artificial, as in the real atmosphere even the driest desert is connected to regions of finite humidity by horizontal advection. The magenta and red curves in fig. 2 show results using horizontal advective time scales of 3 and 6 days, respectively (the no-advection case would correspond to an infinite advective time scale); again the relaxation is towards a target moisture profile equal to that in the RCE. We see that for the 3-day time scale, multiple equilibria exist only over a narrow range of SSTs. The horizontal advective moistening in this case prevents the atmosphere from becoming sufficiently dry to inhibit deep convection once SST exceeds the RCE value of 301 K by a small increment. On the other hand, for a mixing time scale of 6 days, the multiple equilibria persist up to large SST. In this case, the horizontal advective moistening is too weak to overwhelm the vertical advective drying, and the atmosphere can stay dry enough to inhibit deep convection even for large SST if it is initialized sufficiently dry. These results show that the existence of multiple equilibria is not purely an artifact of the neglect of horizontal moisture advection, although sufficiently strong horizontal moisture advection can eliminate the dry equilibrium (or render it unstable so that it cannot be achieved numerically).

In the cases with horizontal moisture advection, the precipitation is a non-monotonic function of SST, with local maxima near 301K, the value at which the RCE was computed. The solutions in this neighborhood are time-dependent, either periodic or chaotic but with a dominant spectral peak. The periods range from 1-20 h, and maximum excursions of 0.5-10 mmd^{-1}, with the largest oscillations tending to occur for the higher SST values although the dependence of amplitude on SST is not monotonic. We do not understand these oscillations in detail and do not address them further here; our interest is in the presence or absence of multiple equilibria.

3.2. Interpretation

We consider under what conditions the multiple equilibria can exist. We can derive some general constraints that any model must obey if it is to obtain a dry solution under boundary conditions and forcings that also allow a rainy solution.

Starting from the primitive equations, the steady temperature and moisture equations in pressure coordinates are

$$\mathbf{u} \cdot \nabla T + \omega \partial_p s = Q_c + R - \partial_p \overline{\omega' s'}, \tag{1}$$

$$\mathbf{u} \cdot \nabla q + \omega \partial_p q = Q_q - \partial_p \overline{\omega' q'}, \tag{2}$$

where $\mathbf{u}$ is horizontal velocity, T temperature in energy units (i.e., multiplied by the heat capacity of air at constant pressure C_p), s dry static energy, q specific humidity in energy units (i.e., multiplied by the latent heat of vaporization L_v), R radiative heating, Q_c convective heating, Q_q

convective moistening, ω large-scale vertical velocity, and ∇ the horizontal gradient on pressure surfaces. $\overline{\omega' s'}$ and $\overline{\omega' q'}$ are the turbulent fluxes, limited to the atmospheric boundary layer (ABL), as parameterized by the model's boundary layer scheme; in the free troposphere these fluxes are incorporated into Q_c and Q_q.

The WTG approximation requires neglect of horizontal temperature advection in the free troposphere; for the sake of argument let us also neglect horizontal moisture advection and horizontal temperature advection in the ABL. If there is no deep convection, $Q_c = Q_q = 0$, the equations for the free troposphere are:

$$\omega \partial_p s = R, \tag{3}$$

$$\omega \partial_p q = 0. \tag{4}$$

Therefore, unless the radiative cooling R is zero, the vertical velocity ω is non-zero and $\partial_p q = 0$. Integrating from the top of the atmosphere, this in turn yields $q = 0$: the free troposphere has to be dry for the non-convective equilibrium to exist. If horizontal moisture advection is included, the free-tropospheric humidity can be non-zero in this dry state.

Equations (3) and (4) can be integrated over the ABL:

$$\langle \omega \partial_p s \rangle = \langle R \rangle + H, \tag{5}$$

$$\langle \omega \partial_p q \rangle = E, \tag{6}$$

where H and E are the surface sensible and latent heat fluxes, and $\langle \rangle$ indicates the integral from the surface to the top of the inversion $p_{\rm inv}$.

In our formulation, ω varies linearly from p_{inv} to the surface. Equations (5) and (6) can therefore be rewritten:

$$\omega_{\text{inv}}(s_M - s^+_{\text{inv}}) = \langle R\rangle + H, \tag{7}$$

$$\omega_{\text{inv}} q_M = E, \tag{8}$$

where the subscript M indicates the vertically averaged value over the ABL and s^+_{inv} is the dry static energy at the top of the inversion. We used the fact that the humidity at that level is zero.

Equation (8) gives us one constraint that must be satisfied in order for the dry solution to occur: the ventilation of the ABL by the subsidence has to compensate the evaporation. On the other hand, Equation (7) shows that the warming by surface sensible heat flux and subsidence has to be compensated by the radiative cooling. The two equations can be rewritten as a single condition for the existence of the non-convective equilibrium:

$$-\frac{\langle R\rangle + H}{s^+_{\text{inv}} - s_M}\, q_M = E. \tag{9}$$

As the SST increases, s_M increases, while s^+_{inv} is fixed in WTG. The ABL stability $s^+_{\text{inv}} - s_M$ thus decreases. The radiative cooling $-\langle R\rangle$ increases too, and the surface heat flux H is not very sensitive to SST changes. The subsidence ω_{inv} therefore must increase with increasing SST. The ABL specific humidity q_M also increases with SST. So the effect of ventilation increases with SST.

To maintain the dry solution over an ocean surface, it is important that the ABL air be able to stay relatively moist as the SST increases so that E is limited and the ventilation $\omega_{\text{inv}} q_M$ is efficient. Otherwise E will become very large due to the large air-sea humidity contrast (taking surface wind fixed), and in general there is no mechanism for R nor $(s^+_{\text{inv}} - s_M)^{-1}$ to become large at the same time to compensate. In nature (and in many models), the ABL generally does stay moist under a dry free troposphere, e.g., in subtropical trade wind regions. In some models, though, this may not occur; obvious examples are idealized models in which the shape of the entire vertical structure of the humidity field is fixed, so that the ABL moisture is proportional to the free-tropospheric moisture. This happens, for example, in the first quasi-equilibrium tropical circulation model (QTCM; *Neelin and Zeng* [2000]; *Zeng et al.* [2000]). This can be remedied by allowing a separate degree of freedom for boundary layer moisture, as in some otherwise similar models [*Wang and Li*, 1993; *Neggers et al.*, 2006; *Sobel and Neelin*, 2006; *Khouider and Majda*, 2006].

The second constraint that must hold in order for the dry solution to occur is that the convective heating and moistening remain zero, or at least small. For a dry free troposphere and a cold sea surface, so that CAPE is negative, this will occur for any reasonable convective parameterization. For a dry free troposphere and a warm sea surface, so that non-entraining CAPE is significantly positive, the results may be model-dependent. Parameterizations which are insufficiently sensitive to dry free tropospheric air may be able to generate some heating. Once heating occurs, if $Q_c + R > 0$ there will be ascent, which will moisten the troposphere, leading to the establishment of the rainy solution. Alternatively, if the scheme is not able to generate deep convection and associated heating, but can produce enough moistening above the ABL to eventually allow deep convection to occur, this will also lead to the rainy solution. It is reasonable to expect that under the same boundary conditions, some models will have multiple equilibria and others will not.

We have performed preliminary experiments with a small number of other SCMs in addition to the GEOS5. In these experiments, we have been able to produce multiple equilibria in some but not all of the models. However, our search over the space of initial and boundary conditions in these other models has been cursory, so it is premature to draw any conclusions from these results. We will report on them in more detail when we have performed a more thorough study.

4. Conclusions

We have shown that a single-column model using essentially the same physics and numerics as a state-of-the-art GCM has multiple equilibria, when run with fixed free tropospheric temperature and diagnostic large-scale vertical velocity according to the weak temperature gradient approximation. When the boundary conditions are such as to allow a rainy equilibrium state, a second equilibrium with no precipitation and a very dry free troposphere also exists, and is reached by initializing the model with a very dry sounding. This dry equilibrium state persists in the presence of parameterized horizontal moisture advection, represented as a relaxation of the specific humidity back to a relatively moist reference profile, as long as the relaxation time scale is not too short. The critical value of this time scale for existence of multiple equlibria is about four days in this model. When the SST is sufficiently low, compared to that at which the free-tropospheric temperature profile would be in radiative-convective equilibrium (RCE), only the dry equilibrium exists.

The existence of the dry equilibrium under SST greater than the RCE value requires that the radiatively-driven large-scale descent be able to export sufficient moisture to balance the surface evaporation, and that the convective parameterization (or explicit convection, in a cloud-resolving model) be sufficiently sensitive to free-tropospheric moisture that the dry troposphere inhibits deep convection from occurring. Whether these requirements are met may be model-dependent.

The existence of these multiple equilibria is a consequence of the interaction between deep convection and large-scale dynamics, with the latter parameterized here through the weak temperature gradient approximation. The multiple equilibria presented here are in this respect fundamentally different than those found by *Rennó* [1997], whose model did not include a representation of large-scale dynamics. It seems possible, but is not obvious, that analogous multiple equilibria would exist for other parameterizations of large scale dynamics, similar in spirit to the WTG approach used here but differing in detail (e.g., *Bergman and Sardeshmukh* [2004]; *Mapes* [2004]; *Kuang* [2007]).

The existence of these multiple equilibria strikes us as a direct and simple expression of the tropical atmosphere's inherent nonlinearity. It illustrates the complexity that is hidden behind the averaging when the time-mean precipitation, for example, is considered as a function of boundary conditions.

We cannot be certain that the multiple equilibria would exist for a hypothetical model with "correct" physics. Recent studies suggest, however, that the tendency of many current convective parameterizations is to have too little, rather than too much, sensitivity to free tropospheric moisture, due to insufficient entrainment (e.g., *Derbyshire et al.* [2004]; *Kuang and Bretherton* [2006]; *Biasutti et al.* [2006]). As we expect that more sensitivity will make the multiple equilibria more likely to exist, this suggests that they are not purely an artifact of a bias in the GEOS5 physics. In addition, in recent simulations of radiative-convective equilibrium on a large domain over uniform SST [*Bretherton et al.*, 2005] deep convection occurs only in a single, small region, while the rest of the domain becomes extremely dry. This behavior is analogous to what we find here, with our single column representing either the dry or rainy region separately. This suggests that a cloud-resolving model run in WTG mode might well exhibit multiple equilibria.

Acknowledgments. We thank David Neelin for a very helpful discussion. This work was supported by NASA grant NNX06AB48G S01.

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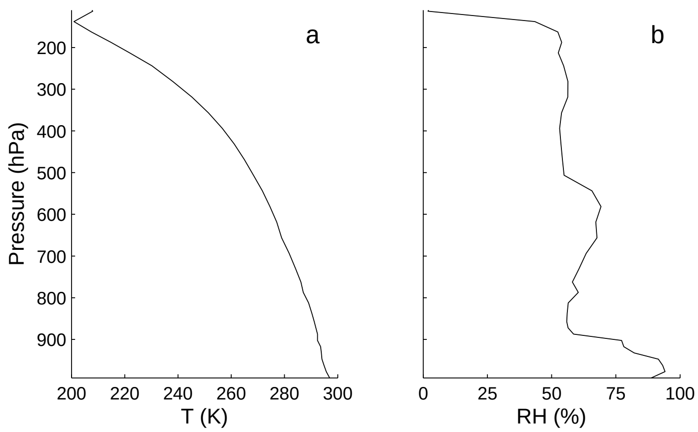

Figure 1. Temperature (a) and relative humidity (b) profiles in the radiative-convective equilibrium state over an SST of 301K. Temperature is in $^\circ K$, relative humidity in %.

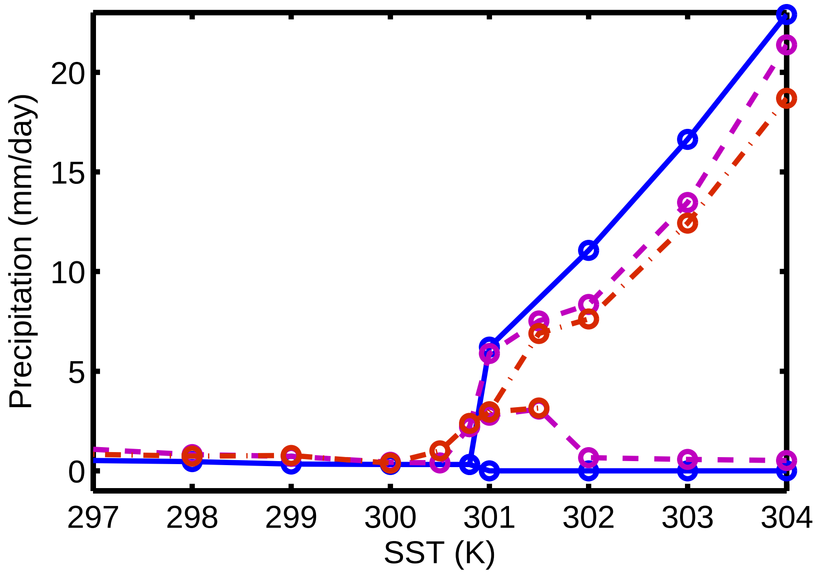

Figure 2. Precipitation (mmd^{-1}) as a function of SST in the convective and non-convective equilibria, with no horizontal moisture advection (blue, solid) and with horizontal moisture advection parameterized as a relaxation back to the RCE profile with a time scale of 6 days (magenta, dashed) and 3 days (red, dot-dashed). Each circle represents a separate model integration.

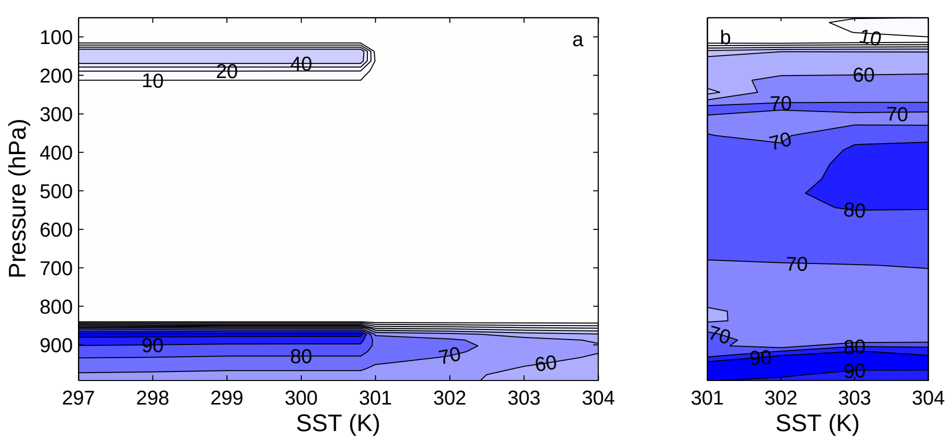

Figure 3. Relative humidity (%) as a function of SST and pressure in the non-convective (a) and convective (b) equilibria, with no horizontal moisture advection.